A new dry hypothesis for the formation of martian linear gullies


Abstract

Long, narrow grooves found on the slopes of martian sand dunes have been cited as evidence of liquid water via the hypothesis that melt-water initiated debris flows eroded channels and deposited lateral levées. However, this theory has several short-comings for explaining the observed morphology and activity of these linear gullies. We present an alternative hypothesis that is consistent with the observed morphology, location, and current activity: that blocks of CO₂ ice break from over-steepened cornices as sublimation processes destabilize the surface in the spring, and these blocks move downslope, carving out levéeed grooves of relatively uniform width and forming terminal pits. To test this hypothesis, we describe experiments involving water and CO₂ blocks on terrestrial dunes and then compare results with the martian features. Furthermore, we present a theoretical model of the initiation of block motion due to sublimation and use this to quantitatively compare the expected behavior of blocks on the Earth and Mars. The model demonstrates that CO₂ blocks can be expected to move via our proposed mechanism on the Earth and Mars, and the experiments show that the motion of these blocks will naturally create the main morphological features of linear gullies seen on Mars.

Keywords:
Mars, Surface
Geological processes
Ices

1. Introduction

Observations of long, shallow, and narrow features eroded into the lee-slope of a mega-dune in Russell crater (Fig. 1A) were first reported by Mangold et al. (2002). Further imaging by the Mars Orbiter Camera (MOC) on the Mars Global Surveyor and the High Resolution Imaging Science Experiment (HiRISE) on the Mars Reconnaissance Orbiter has demonstrated that these linear gullies (terminology adopted within recent literature [Dundas et al., 2012; Védie et al., 2008]) to differentiate these features from gullies of alcove–channel–apron [ACA] form) are found within many dune fields and on sandy crater walls within the mid-latitudes on pole-facing slopes (Di Achille et al., 2008; Reiss et al., 2007). These slopes typically range from 7° to 12° (well below the angle at which a dry granular material is expected to flow [Jouannic et al., 2012; Mangold et al., 2003; Reiss et al., 2007]), but the gullies alcoves and grooves appear to originate within the steeper upper slope (can be >25°: Jouannic et al., 2012). Over the past three Mars years, HiRISE images show that existing grooves have elongated and new grooves have formed at the start of each spring (Dundas et al., 2012; Reiss et al., 2010; this study), demonstrating that these features are active in the present-day martian climate.

As shown in Fig. 1, a linear gully on Mars consists primarily of a long (few hundred meters up to 2.5 km) groove that has near-uniform width (generally a few to 10 m wide, and sometimes with slight narrowing downslope), is near-linear throughout most of its length (but sometimes contains zones of low-to-high sinuosity), and is commonly surrounded by levées. Groove incision depth is usually from less than 1 m to 2 m and appears shallowest within regions of lowest slope (at the base of the dune), but some portions can exceed 3 m (Jouannic et al., 2012). The groove is generally topped by a small alcove and/or converging small grooves that originate at the dune brink (or, if on a sandy slope, where sand first becomes visually apparent). Downslope, the groove abruptly ends and lacks a debris apron. It sometimes ends with a terminal pit, sometimes in a chain of pits, or sometimes in a series of divergent small grooves, each with a terminal pit. A pit generally has a diameter comparable to, but larger than the groove width. Grooves sometimes converge downslope. Except within the very distal portion, they do not diverge.
Many studies have attempted to explain the mechanism(s) responsible for formation of these features. The most common hypothesis is that linear gullies are formed by water-supported debris flows (e.g., Costard et al., 2002; Gargani et al., 2012; Jouannic et al., 2012; Mangold et al., 2003; Mangold et al., 2010; Miyamoto, 2004; Reiss and Jaumann, 2003; Reiss et al., 2010; Védie et al., 2008) based on proposed morphological resemblance to terrestrial landforms carved by flowing water. Dry hypotheses have also been proposed: based on correlations observed between defrosting markers and gullies in Russell crater, Hansen et al. (2007) proposed that sublimation of CO₂ could play a role in linear gully activity. Di Achille et al. (2008) examined pits found on Noachis and Aonia Terra dunes and suggested that defrosting processes, glacial-like creep, and rolling sand–ice aggregates may form these features.

In this paper, we present a new and detailed look at a dry hypothesis: that seasonal CO₂ ice forms in the winter and breaks into blocks that fall down the dune slope, carving out a groove and leaving a terminal pit when the block comes to a rest and sublimates. First, we discuss some of the shortcomings of the debris flow theory (Section 2). We then describe, in detail, the dry hypothesis (Section 3). We present analogue and experimental evidence that the main morphological features, placement, and activity of martian linear gullies can form through interactions between solid blocks of CO₂ ice with a granular surface and without the need for liquid water (Section 4). We demonstrate consistency between the morphology predicted by the proposed mechanism and both analogue features formed by falling blocks on a range of surfaces and experiments involving dry ice blocks moving down terrestrial dune slopes. We also report on observations of blocks that briefly appear within linear gullies on Mars. We discuss a theoretical model of the mechanism by which CO₂ ice is able to levitate and move down shallow slopes (Section 5) and use this to scale relevant forces between Earth and Mars conditions.

2. Debris flow hypothesis

Debris flows consist of a gravity-driven mass of poorly sorted sediment, dispersed within a fluid slurry (Iverson, 1997; Takahashi, 1981). Both solid and fluid forces strongly influence the motion of the flow as the pore fluid mediates inter-granular friction and collisions but do not completely suspend the sediment, thus distinguishing debris flows from related phenomena such as dry granular flow, rock avalanches, turbidity currents, and sediment-laden water floods. The resultant geomorphological feature is generally composed of a source alcove, a single linear channel with lateral ridges (levées), and a terminal distal cone or fan. These features can also exhibit pervasive, fluid-like deformation such as the movement of even boulder-rich debris through tortuous channels, across gentle slopes, and around obstructions (Iverson et al., 1997).

The debris flow hypothesis was originally applied to martian linear gullies by Costard et al. (2002) based on morphological analogy between these martian landforms and debris flows in Greenland and following a suggestion by Malin and Edgett (2000) that ACA gullies found on crater walls were formed due to groundwater seepage and surface runoff. Costard et al. (2002) suggested the linear gullies (and ACA gullies) formed during a previous period of high obliquity, when water ice may have accumulated in large volumes within the near-subsurface at the top of the dune and then melted. Subsequent studies have since examined this meltwater-based formation mechanism (e.g., Mangold et al., 2010; Miyamoto, 2004) in efforts to quantitatively connect observed morphology to fluid metrics, and some have even hypothesized that debris flows could occur under the present climate conditions due to episodic or seasonal melting of water ice (Reiss and Jaumann, 2003; Reiss et al., 2010).

Many observations, experiments, and theoretical models, however, do not support the debris flow hypothesis. In particular:

1. Martian linear gullies have two salient morphological features that are unexplained by the debris flow theory: the absence of depositional aprons at the terminus and presence of meandering patterns (Fig. 1C). Some studies have hypothesized complicated mechanisms to account for these features, such as a progressive increase in water-fraction downslope (Gargani et al., 2012; Jouannic et al., 2012) or pulses of water over ice-rich permafrost (Védie et al., 2008). However, laboratory and numerical models involving simulated debris flows on dune slopes have generally been unable to reproduce these morphologies (e.g., Coleman et al., 2009; Conway et al., 2011; Mangold et al., 2010). Moreover, naturally-occurring terrestrial debris flows on cold-climate aeolian dunes do not yield meandering channels and do form well-defined terminal lobes or aprons (e.g., Bourke, 2005; Hugenholtz et al., 2007; Hooper et al., 2012). Thus, it remains unexplained how debris flows on a
dune in the Russell crater, for example, would be capable of translating down a gentle slope for 2 km, carving out grooves of near-uniform width without depositing sediment at the terminus. The balance of erosion and levee deposition implied by this hypothesis is extraordinarily precise.

(2) Theoretical studies have explored how water could erode a slope and produce these features (Jouannic et al., 2012; Mangold et al., 2003; Mangold et al., 2010; Miyamoto, 2004), but none have conclusively connected these models with a viable and sufficient water source in the current climate. Estimates of seasonal water frost that can accumulate from the atmosphere (a layer tens to hundreds of microns thick (Vincendon et al., 2010), corresponding to 1 m$^3$ in a notional 100 × 100 m source area) are significantly lower than the smallest required water volume estimates (e.g., 100 m$^3$; Miyamoto, 2004).

(3) Linear gully formation activity (such as 1–2 m wide grooves that extended 50–120 m with an interval of a few weeks; examples are shown in Fig. 3) has been observed over the past three Mars years during early spring (Dundas et al., 2012; Reiss et al., 2010; this study). Possible water ice melt and brine solutions have been proposed (Reiss et al., 2010; Kereszturi et al., 2009). However, although it is theoretically possible to achieve brine eutectic temperatures as low as 145 K (the CO$_2$ frost point) for vanishingly small quantities of liquid (Chevrier and Altheide, 2008; Rivera-Valentin et al., 2011), the minimum confirmed (measured) eutectic temperatures are no lower than 200 K (Chevrier and Altheide, 2008; Möhlmann and Thomsen, 2011). We have observed that the upper slopes on which these gullies are found were still covered by CO$_2$ frost when activity was occurring (e.g., Fig. 3), and thus both the surface and atmosphere were far too cold for liquid water or brines to be stable. Dark patches at the surface may become warmer than the CO$_2$ frost point (Möhlmann and Kereszturi, 2010), but then the local relative humidity is too low for brine stability (D. Möhlmann, personal communication, 2011). Freezing point depression at particle-ice interfaces in the shallow subsurface is possible (Hansen-Goos et al., 2012), but in layers only a few molecules thick that cannot easily mobilize debris (Kossacki and Markiewicz, 2010).

(4) The common presence of terminal circular depressions or pits on many linear gullies (Fig. 1D) is problematic. We found no reports of pits occurring at the distal end of terrestrial debris flows, so their formation seems unlikely to be related to the debris flow mechanism. These features have been hypothesized as uniquely forming on Mars due to sublimation of incorporated volatiles within a debris flow (Védié et al., 2008), but this is inconsistent with their formation only at the terminus (e.g., not within the levees) and the lack of a debris apron containing the remainder of the eroded materials. Reiss et al. (2010) suggested that they were detached older flow fronts, but this is ruled out by observed pit formation (Dundas et al., 2012).

Given these inconsistencies and considering the context of present-day atmospheric conditions on Mars, we propose an alternative hypothesis that can create the observed morphology of these features without requiring water.

3. The CO$_2$ blocks hypothesis

A correlation between seasonal features and gullies of all types has led many to propose that seasonal defrosting may relate to or even directly cause gully formation: Bridges et al. (2001) had noted that dark spots were found preferentially within ACA gullies found on crater slopes and Mangold et al. (2003) observed the same relation within the linear gullies found on the Russell crater megadune. These studies, however, attributed gully formation to the melting of water ice—a hypothesis that, as described above, does not appear to be consistent with current observations. Instead, sublimation of CO$_2$ is more likely to play a role in linear gully activity (Di Achille et al., 2008; Hansen et al., 2007).

Beginning in the martian autumn, temperatures drop and up to 30% of Mars’ CO$_2$ atmosphere condenses (Leighton and Murray, 1966), forming frost on all surfaces located poleward of 60°S and many surfaces through 33°S (Schroghofer and Edgett, 2006), preferentially accumulating within areas of high shadow (such as within grooves and alcoves) and on pole-facing slopes (Gardin et al., 2010). Though some atmospheric water does freeze and accumulate on the martian surface (Kereszturi et al., 2009; Vincendon et al., 2010), it forms deposits maximally tens of microns thick (<1 mm).

The vast majority of martian seasonal frost is made up of CO$_2$. Compaction and crystal-sintering decreases porosity and increases the density of the CO$_2$ frost layer (Matsuo and Heki, 2009)—in the polar regions, the initially fluffy (100 kg m$^{-3}$) layer will compact into slabs of translucent ice (1600 kg m$^{-3}$; Kieffer, 2007). This results in a peak CO$_2$ ice layer about 1–2 m thick in the polar regions and a more porous layer up to a few tens of centimeters thick within the mid-latitudes (Aharonson et al., 2004; Kelly et al., 2006; Matsuo and Heki, 2009; Smith et al., 2001). We suggest that this seasonal CO$_2$ layer is likely to have maximum thickness within the regions of preferential accumulation (e.g., such as pole-facing slopes and within alcoves), consistent with observations of Russell crater by Gardin et al. (2010) who found that the strongest CO$_2$ ice signatures at the end of winter were located along the brink of the megadune.

As spring approaches, the CO$_2$ begins to sublimate and the seasonal polar cap recedes. We propose that coherent blocks of CO$_2$ ice break off and fall from oversteepened cornices and then slide onto the lower slopes. This is consistent with observations of blocks in linear gullies, as reported by Dundas et al. (2012). As we will show with our experiments, despite the low slopes, lubrication from sublimation can allow these blocks to slide long distances downslope. As the blocks slide, they erode a shallow groove by pushing unconsolidated material forward and to the sides, forming levees. This displacement may be enhanced by sublimation throwing material to the sides of the block, perhaps accounting for dark halos seen around grooves that recently formed and/or temporarily contain a frost block (Fig. 3). If the block encounters small topographic perturbations, such as the 2–4 m wavelength ripples common on martian dunes and other sand-covered surfaces, it can be deflected, giving rise to meanders in its path. Alternatively, or perhaps in addition, an irregularly shaped block can pivot as it moves downslope, resulting in a slightly-wavy path.

The block will continue to sublimate after it came to rest, kicking sand out from beneath it and eventually disappearing—thus distributing sand to its sides and leaving a terminal pit. If the block breaks apart as it moves downslope, due to destabilization through impact with the ground and/or sublimation, then these pieces can be strewn. If all pieces move in the same direction, then a chain of pits can be formed (Fig. 1D). If the pieces move in different directions, then thin branching tracks, each with its own block/eventual pit, can be formed.

4. Experimental, analogue, and observational evidence

In this section, we present evidence supporting the connection between our hypothesized mechanism and the observed morphology of martian linear gullies. We focus primarily on a series of experiments done with dry ice blocks on dunes in Utah (small
barchans in Sand Hollow State Park and large barchans in Coral Pink Sand Dunes National Monument. Although these experiments were not conducted at martian conditions (i.e., the dune sand surface was ~90°F and under terrestrial atmospheric conditions and gravity), these experiments demonstrate clearly that the types of morphologies that we observe on Mars can be created by sliding CO2 blocks, at least at a small-scale, whereas experiments and simulations that involve seepage and runoff of liquid water on sandy slopes do not produce morphologically-comparable forms (e.g., Coleman et al., 2009; Conway et al., 2011; Mangold et al., 2010). Individual block dimensions and brief experiment descriptions are given in Table 1. In the following sections, we group together experimental results based on the resultant observation type. We also discuss some Earth and Mars analogue and Mars observational evidence, as appropriate.

4.1. Block appearance

We exposed water ice and CO2 ice slabs on dunes (Fig. 2a) and observed changes in block appearance as they disappeared and interacted with their sandy surroundings. Whether stationary or pushed along the surface of the dune, the water ice block surface became wet and then dirty in appearance as a sand covering stuck to the exposed surfaces of the block. After sand saltated over one block for about an hour, a 1–2 mm covering of wet sand completely obscured the surface. The CO2 block, however, remained pristine in appearance. If a thin covering of sand was placed onto or blown onto the block, the sand was quickly dislodged by any movement of the block or particularly large gusts of wind as continual sublimation pushed sand grains away. When a CO2 ice slab was placed on the ground, sublimation along its base caused sand to jump away (and in one case, to appear to “boil”) and the block would dig itself slightly into the dune surface (Fig. 2b). Small channels also formed along the underside of the CO2 block and within the sand as the sublimating gas created efficient pathways for escape (Fig. 2c).

Blocks interpreted as CO2 ice blocks have been observed within linear gullies on Mars. A monitoring campaign over active gullies (Dundas et al., 2012) observed 1–2 m wide and long blocks (thickness cannot be determined) within linear gullies at the end of the last two winter seasons. These blocks are visually very bright relative to their surroundings. Examples of frost blocks observed during the most recent Mars winter are shown in Fig. 3. Similar examples from the previous winter were described in Dundas et al. (2012): four blocks appeared in early spring in Mars Year 30 (using the calendar of Clancy et al. (2000); the start of MY 31 and the southern autumnal equinox was on September 13, 2011) within a dune field at 70°S. In the Mars Year 30 examples, the dune slope lacked any clear pre-existing dune grooves, but did contain degraded (but still active) ACA dune-gullies. The blocks were at the end of new narrow lineations, making it fairly apparent that the block moved downslope and marked the surface. In the Mars Year 31 examples (Fig. 3), the blocks appeared within pre-existing grooves and neighboring regions contained newly formed channels and pits. In both years, dark haloes appeared around most of the blocks and new grooves that appeared to extend over the levées—perhaps due to sand being thrown to the sides as the block sublimates. These blocks decreased in size and completely disappeared during the late spring or summer. The location, visual appearance, and timing of (dis)appearance of these blocks indicate that they are composed of seasonal ice and CO2 is the most plausible material for producing blocks of this scale.

4.2. Groove formation

As shown in Figs. 4 and 5 (and SOM videos 1–3), rectangular CO2 blocks that slid down a dune slope created both straight and slightly-wavy grooves with near-constant width. In these experi-

<p>| Table 1 Summary of experiments done on terrestrial dunes with ice blocks. All dimensions measured to nearest 3 mm or 10 g. |
| --- | --- | --- | --- | --- |</p>
<table>
<thead>
<tr>
<th>Experiment</th>
<th>Type</th>
<th>Block</th>
<th>Size (mm)</th>
<th>Mass (g)</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left on surface</td>
<td>Water</td>
<td>W1</td>
<td>140 × 140 × 25</td>
<td>490</td>
<td>Left a very wet spot, but no other surface change</td>
</tr>
<tr>
<td>CO2</td>
<td>D1</td>
<td>102 × 51 × 32</td>
<td>300</td>
<td>Disappeared in 1.9 h. Carved out shallow pit (&lt;3 mm depth) of comparable surface area as the original block</td>
<td></td>
</tr>
<tr>
<td>CO2</td>
<td>D2</td>
<td>102 × 51 × 38</td>
<td>410</td>
<td>Disappeared in 2 h. Carved out a shallow pit (~7 mm depth) of comparable surface area as the original block</td>
<td></td>
</tr>
<tr>
<td>CO2</td>
<td>D3</td>
<td>108 × 102 × 32</td>
<td>540</td>
<td>Disappeared in 2 h. Carved out a shallow pit (~10 mm depth) of comparable surface area as the original block</td>
<td></td>
</tr>
<tr>
<td>CO2</td>
<td>D7</td>
<td>Not measured</td>
<td></td>
<td>Picked up a few times, looking for morphological changes in ice/sand surface. Small (2–4 mm wide) channels formed in both ice and sand surface after a few minutes</td>
<td></td>
</tr>
<tr>
<td>Buried below surface</td>
<td>Water</td>
<td>W2</td>
<td>140 × 140 × 25</td>
<td>500</td>
<td>Buried ~25 mm below surface. Left a pit of size comparable to the original ice block and a wet surface</td>
</tr>
<tr>
<td>CO2</td>
<td>D4</td>
<td>Not measured</td>
<td>240</td>
<td>Buried ~20 mm below surface. Surface directly above the block (and the floor of the collapsing pit) was frosted until the block had completely disappeared. Pit collapsed at least 13 mm and had area comparable to the original block</td>
<td></td>
</tr>
<tr>
<td>CO2</td>
<td>D5</td>
<td>203 × 76 × 57</td>
<td>1080</td>
<td>Buried in pre-existing hole and covered with ~45 mm of sand, then the surface was smoothed. No pit was seen to form, but a gas cavity appeared to exist below the surface (as determined by poking the ground with the thermometer and feeling the change in resistance)</td>
<td></td>
</tr>
<tr>
<td>Run down slope</td>
<td>Wood</td>
<td>C1</td>
<td>267 × 178 × 51</td>
<td>2000</td>
<td>Lee slope—pushed, but block did not move (SOM video 5)</td>
</tr>
<tr>
<td>Water</td>
<td>W3</td>
<td>254 × 152 × 45</td>
<td>1350</td>
<td>Lee slope—pushed, but block did not move (SOM video 5)</td>
<td></td>
</tr>
<tr>
<td>CO2</td>
<td>D9</td>
<td>210 × 203 × 51</td>
<td>3100</td>
<td>Lee slope—pushed, and block slid down full slope (14 m) in &lt;4.5 s, ploughing through ripples/sand obstacles and partially burying self upon impact into sand at dune base (SOM video 3)</td>
<td></td>
</tr>
<tr>
<td>CO2</td>
<td>D6</td>
<td>203 × 102 × 57</td>
<td>1400</td>
<td>Lee slope—initially balanced and horizontal on dune brink (with COM behind brink). It tilted, but did not fall</td>
<td></td>
</tr>
<tr>
<td>CO2</td>
<td>D9</td>
<td>Not re-measured</td>
<td>2850</td>
<td>Stoss slope—10 short runs with same starting point/orientation. See text for full description</td>
<td></td>
</tr>
<tr>
<td>CO2</td>
<td>D8</td>
<td>159 × 140 × 51</td>
<td>1100</td>
<td>Stoss slope—3 long runs, ran out full slope (73 m) in 29 s, stopping after hitting vegetation (SOM video 1)</td>
<td></td>
</tr>
</tbody>
</table>
ments, a rectangular slab (dimensions given in Table 1) was placed on the dune slope. The slab would not slide if just placed on the surface, but a very slight push (such as a couple of fingers pushing on the block or a gentle nudge) was enough to initiate movement—likely comparable or of less force than that imparted to a block fallen from the dune brink. The block carved out a groove by

Fig. 2. (a) Example CO₂ (left) and water (right) ice blocks used in experiments. Notice that the CO₂ block is sublimating, generating a constant cloud of CO₂ vapor from the top and sides. This sublimation keeps the block pristine white as sand cannot become stuck to the sides of the block (as is visibly the case with the water block). The sublimation can be very vigorous, causing sand to be pushed away from the bottom and sides of the block. An example of particularly strong sublimation activity is shown in (b), where the sand appeared to “boil” around the edges of the block, which had been slid down the slope and then stopped mid-slope by a shoe (arrows highlight some of the larger bubbles of sand that were forming). This sublimation can also carve out shallow channels in sand (c) and the ice’s surface (not shown as difficult to photograph). In (c), white arrows highlight the levées that formed as sublimation caused the block to dig itself into the sand. The black arrows highlight channels carved into the sand as the gas escaped. Before taking this picture, this CO₂ block (D7 in Table 1) was left on the surface for about 10 min.

Fig. 3. Images showing recent linear gully activity and likely frost blocks within Matara crater dune field (49.4S, 34.7E). White arrows point out the frost blocks, which appear very bright against the mostly defrosted dune surface. Black arrows highlight new channels and terminal pits. In addition to the new channels and terminal pits, dark and bright haloes are visible around the active regions (and these haloes, like the blocks, disappear as surface finishes defrosting). Images b and e are of ESP_013834_1300, during early summer in Mars Year 29; images a, c, and f are of ESP_029038_1305, just at the start of spring in Mars Year 31; and images d and g are of ESP_029961_1305, at mid-spring in Mars Year 31. (Note that some of the changes in apparent depth of features between images are due to changes in illumination conditions.) The scale bar shown in image b applies to images b–g.
pushing sand towards the sides, eroding material along the base of the groove and forming shallow levées (Fig. 4). Once the block began moving, it would slide down the slope unassisted and could pick up enough velocity to plow straight through small-scale topographic features (e.g., ripples, footprints; Fig. 4) that were a few times taller than the slab thickness, spraying a small amount of sand (SOM videos 1, 3 and 4), but otherwise not changing direction or resultant groove depth and levée height. As the block generally came to rest after running into soft sand at the base of the lee slope or after running into vegetation, there was no depositional apron. However, even when a block was stopped mid-slope (e.g., by encountering a large obstacle or a damp patch of sand), there was little sand deposited in front of the block.

Although neither a water ice block nor a wooden plank of similar dimensions would move at all down even the steep lee slope (SOM video 5), the CO2 ice blocks slid easily on both the lee slope (assumed to be at angle of repose ~33°; at least 20°) and on the upwind slope (average angle ~6°, estimated from dune dimensions) (SOM videos 1–4). As long as the surface sand was dry (which was the case for all Utah experiments; some patches of slope seemed slightly damp in later experiments at Kelso dunes, CA), the runs extended the full length of the slope, ending upon impact into soft sand at the base of the lee slope (when the block would be partially buried) or after encountering several small bushes that grew in the interdune plain (at the base of the stoss slope). Average downslope velocity was 3.1 m s\(^{-1}\) on the lee slope (traversed distance of 14 m) and ~2.5 m s\(^{-1}\) on the upwind slope (73 m). In a latter experimental trial at Kelso Dunes, a dry ice block comparable in size to a block used in Utah (D6 in Table 1) ran down the full slope (~20° on average) of the tallest dune—about 180 m! This block carved out a shallow trough even over lower slopes (some portions were <10°) and slightly damp sand patches (although the track was shallower; the damper patches did stop a couple of dry ice blocks midslope). The block went much slower than the Utah experiment runs (on average <1 m s\(^{-1}\), but with a range of velocities as local sand conditions changed over its course—velocities comparable to other runs were achieved over steeper or drier sand), but the block still managed to flatten shallow ripples and move through shallow footprints.

To evaluate the effects of activity that was repeated seasonally, we did multiple block slides in the same location. We slid one CO2 block (D9 in Table 1) 10 times down the stoss slope of a dune at Coral Pink Sand Dunes. Each run was started in the same location and with the same slab orientation. All runs went roughly straight downslope, along the same track, but there was some path widening and divergence. Within the upper portion of the track (where the slab was picking up velocity), the block would pivot and bounce around a bit off of small-scale topography on the slope—including the levées laid down by previous runs (initially < 1 cm in height above the undisturbed surface; increased to almost

![Fig. 4](image-url) Example images of the groove carved out by a sliding CO2 block: (a) the starting point of the multiple runs. The deepest trough/tallest levées were formed in this region. (b) A groove that went through a large footprint (of similar width and depth as the width and thickness of the ice block). Note that the path was not deflected in this case. (c) Two tracks. The blocks were started at the same location much further up the hill and followed the same general gradient, but had diverged slightly in this region.

![Fig. 5](image-url) Example of divergent tracks and meandering grooves that formed at the top of the slope during the multiple run experiment.
ice can form levéed grooves in dry unconsolidated, granular blocks on terrestrial dunes and on block behavior on Mars. We also investigate blocks were placed onto or beneath the dune surface and that falls from the crest of a dune similar to the block dimensions, and with depth increasing as the 

thickness and new grooves in Russell crater have also appeared to the thin and new grooves in Matara crater (49°C). Although the narrower features of- 

tation (providing strong evidence that the block was suspended and motion was occurring over a lubricating gas cloud). In other instances, these pivots would affect the block’s course, resulting in small-amplitude meanders in its track. After the slab picked up velocity, it would head mostly straight downslope but would veer towards one of two tracks. The block initially alternated equally between the two tracks, but the last three runs all went to the left track. This caused the left track to appear slightly deeper than the right track at the end (but not by a measurable amount).

These experiments clearly demonstrate that a sliding block of CO2 ice can form levéed grooves in dry unconsolidated, granular material. Disconnected/uneven tracks can also be formed by solid blocks tumbling downslope over unconsolidated, granular material (Fig. 6), as the blocks sometimes did if they picked up sufficient velocity (SOM video 3). There are many examples of tracks on Earth and Mars created by boulders rolling down sandy slopes (e.g., Roberts et al., 2012). Some tracks are discontinuous, perhaps due to the skipping motion of the boulders as they rapidly translate downslope, but along lower slope gradients the tracks become more continuous and can contain subtle sinuosity (Fig. 6A–C). Although forming tracks through a slightly different process, rolling terrestrial snowballs also produce similar features (Fig. 6D).

In Russell crater, several grooves (a few meters wide) have lengthened by 50–100 m during the last two Mars winters (Dundas et al., 2012; Reiss et al., 2010) and new thin grooves (~1 m wide) have appeared (Reiss et al., 2010). Grooves comparable in width to the thin and new grooves in Russell crater have also appeared in Matara crater (49°S), sometimes associated with a large terminal pit (Dundas et al., 2012, Fig. 3). Although the narrower features often fade into the background texture of the dune slopes/ripples over the next spring and summer, the similar morphology and common association with a terminal block/pit strongly suggests that all are genetically similar.

4.3. Pit formation

CO2 blocks were placed onto or beneath the dune surface and allowed to sublimate. These generated shallow pits of areal extent similar to the block dimensions, and with depth increasing as the block increased in volume (Table 1). We hypothesize that deeper pits are likely to result when the block is completely buried; however, our one deep burial experiment did not result in any obvious collapse (although a temporarily stable gas cavity was formed beneath the surface; this was detected by pushing a thermometer into the surface and feeling the abrupt difference in resistance – D5 in Table 1).

Further work is needed to quantitatively connect block size and sublimation dynamics with pit formation, but these preliminary observations shows that sublimation of a CO2 block can generate at least a shallow pit. The buried water block also generated a pit as the water melted, leaving a void space. However, the block left at the surface did not create any change to the surface beyond wetting the sand.

5. Model of block-lifting via sublimation

To demonstrate that sublimation of an ice block can produce a layer of vapor that is sufficient to initiate and assist block transport downslope, we first aim to estimate the lift force that can be generated by an ice block that is in contact with warmer sand. We then estimate if this force will be sufficient to levitate the block. Finally, as eventually the sand will be cooled by the block, we estimate the timescale over which the sand will be sufficiently warm to continue lifting the block. By contrasting this timescale against reasonable timescales of transport (via which new and warm sand can be reached), we can evaluate whether downslope transport of a levitating ice block can be sustained.

We apply this model to CO2 blocks on terrestrial dunes and on martian dunes to explain our experiment results and to make predictions of CO2 block behavior on Mars. We also investigate whether water ice blocks could move via this mechanism on Mars (where they might sublimate, rather than melting as the water ice blocks did in the terrestrial experiments).

5.1. Theoretical model for block levitation

We consider a block of CO2 that falls from the crest of a dune onto the dune slope. The surface of the block will be at the frost point $T_f$ whereas the sand will be at some higher temperature $T_s$ that we assume is initially uniform and equal to $T_s$. If the sublimation or vapor pressure at this temperature significantly exceeds the atmospheric pressure then rapid sublimation can occur that is only limited by the heat flux. As there will be a heat flux from the sand to the block, the surface of the block will sublimate and gaseous

![Fig. 6. Examples showing the morphology of grooves produced by tumbling blocks on granular slopes: (a) martian boulder track produced by rolling boulder (15 m diameter), showing deflections due to topography (ESP_015900_1465); (b) relatively straight boulder track on the same slope created by a smaller boulder (5 m in diameter); other boulder tracks along this martian slope (not shown) are formed by even smaller boulders (down the HiRise resolution limit of 1 m diameter); (c) groove created when a small pebble (30 mm diameter) was rolled down a terrestrial sandy slope; (d) grooves created by rolling snowballs.](image-url)
CO₂ will be generated. We assume that this vapor is generated uniformly over the bottom of the block, resulting in a mass flow rate \( q \) perpendicular to the surface which gives a mean gas velocity \( u_0 = q/\rho_g \), where \( \rho_g \) is the density of the gaseous CO₂. Within the sand, the escaping CO₂ gas satisfies Darcy’s law:

\[
vu + k \nabla p = 0,
\]

where \( p \) is the pressure, \( v \) is the gas velocity, \( v \) is the dynamic viscosity, and \( k \) is the sand permeability. For simplicity we neglect the variation of the density with pressure and temperature and assume that the CO₂ is incompressible so that \( \nabla \cdot v = 0 \). At the surface of the sand, pressure is atmospheric pressure \( (p = p_0) \) and \( \nabla \cdot v = u_0 \) where the sand is in contact with the ice block (where \( z \) is the surface normal into the sand).

We consider two shapes of block: (1) the block is rectangular with width \( 2R \) and length \( L \) such that \( R \ll L \) (Fig. 7). This problem can then be approximated as a two-dimensional problem where \( z \) is the vertical coordinate measured downwards into the sand and \( r \) is the coordinate across the block. (2) Alternatively, we consider the three-dimensional problem of a disc of radius \( R \). \( z \) is again the vertical coordinate but now \( r = \sqrt{x^2 + y^2} \) is the radial coordinate parallel to the sand surface. The derivation of these solutions is given in Appendix A; we find that under the block

\[
p = C_u u_0 y / (R^2 - r^2), \quad u_r = u_0 y / (R^2 - r^2), \quad u_z = u_0,
\]

where \( C_u = 1 \) in two dimensions and \( C_u = 2/\pi \) in three dimensions. Integrating this pressure over the surface of the block we get the lift force

\[
F = C_i A \frac{R u_0 y}{k},
\]

where \( A \) is the surface area of the block and \( C_i \) is a numerical factor: in two dimensions \( A = 2RL \) and \( C_i = \pi/4 \approx 0.79 \), in three dimensions \( A = \pi R^2 \) and \( C_i = 4/(3\pi) \approx 0.42 \). Natural blocks, provided they are convex in areal perimeter and planar, are likely to be somewhere between these two shapes and thus should have a proportionality constant \( C_i \) between \( \pi/4 \) and \( 4/3\pi \). This shows that the lift force can be thought of as the product of a geometric factor, the surface area of the block and a pressure \( R u_0 y/k \) that depends on the smallest horizontal dimension of the block. For a block in solid contact with a granular bed, friction must be overcome for the block to move. Friction is proportional to the normal force \( T = \mu N \), where \( N = W\cos \theta - F \) is the normal force, \( W \) is the weight of the block, \( \theta \) is the slope angle and \( \mu \) is the Coulomb friction coefficient. Since the driving force is due to the component of gravity down the slope, the block can start to move if \( W\sin \theta > \mu (W\cos \theta - F) \). Equivalently, if

\[
\tan \theta > \mu - \frac{F}{W\cos \theta},
\]

the lift \( F \) will act to reduce the slope angle necessary for motion. If \( F > W \), motion can start on any slope. Since \( W = g \rho_i H A \), this ratio of lift to weight can be expressed as

\[
\lambda = \frac{F}{W} = \frac{C_i}{R} \frac{u_0 y}{g k \rho_i}.
\]

Now we calculate the expected heat flux into the solid ice. The full time dependent two- or three-dimensional heat equation does not have a convenient solution, but we can find a lower bound for the heat flux by considering the one-dimensional heat equation

\[
\rho_i C_i \frac{\partial T}{\partial t} = \frac{k}{H} \frac{\partial^2 T}{\partial z^2}, \quad T(t, 0) = T_0, \quad T(0, z) = T_0.
\]

where \( \rho_i \) is the bulk density of sand, \( c \) is the heat capacity and \( k \) is the thermal conductivity. Since the atmosphere is primarily CO₂ we assume that a CO₂ block is in equilibrium with the atmosphere so that its surface temperature is the sublimation temperature corresponding to atmospheric pressure \( (T(T_0, 0) = T_s) \). For simplicity we assume that the sand is all initially at a uniform temperature \( (T(0,z) = T_0) \). Eq. (6) then has the solution

\[
T = T_s + (T_0 - T_s) e^{-(z^2)/(4k t / c \rho_i)},
\]

and the heat flux into the ice block is

\[
h(t) = \frac{k}{H} \frac{\partial T}{\partial z} \bigg|_{z=0} = (T_0 - T_s) \sqrt{\frac{k c \rho_i}{\pi t}}.
\]

This combination \( \sqrt{k c \rho_i} \) is known as the thermal inertial.

Now we assume that this heat all goes into sublimating the CO₂ and neglect the subsequent heating of the block and the gas. As \( u_0 = h/\rho_g \), we can use \( h \) to calculate

\[
\lambda = \frac{R}{H} C_i \sqrt{\frac{t_0}{\pi t}},
\]

where \( t_0 = \kappa / (\rho_g c (T_0 - T_s) (h/e_0 k_0 g))^2 \). Thus, we see that as long as \( t > \frac{t_0}{R^2/m^2} \), the block will be lifted and moved by the sublimating gas on any slope.

However, if a block is to continue in its motion, it must move far enough to come into contact with fresh, hot sand before the sand cools and sublimation no longer supports the block’s weight. Assuming complete levitation, a block will move a distance of at least \( (r^2 \sin \theta^2)/2 \) (further if it has initial velocity). Thus it will move for a time duration of \( \tau = \sqrt{4 R/g \sin \theta} \). Comparing this to \( \geq t_0 \) and dropping numerical/geometric factors, we find that the non-dimensional group that determines the mobility of a block is

\[
\frac{t^2}{t^*} \sim \frac{R^2 t_0^2}{H^2 c^2} \sim \frac{R^2 t_0^2 g}{H^2 R} \frac{\kappa (\kappa + p c^2) (T_0 - T_s)^4 v^4}{(e_0 k_0 g)}.
\]

In summary, when this ratio \( > 1 \), then the block is likely to be mobilized. (If however a rectangular block moved along its long axis \( l \) it would be less mobile by a factor \( R/l \) since it will have to move further to come into contact with fresh, hot sand.)

This model is only suitable to discuss the initiation of motion. Once a block starts to rise, other factors become important that are neglected by this model. First the pattern of air flow will change as there will now be a shallow lubrication layer under the block allowing gas to escape more rapidly thus reducing the pressure and the lift force. As the block moves there will also be

![Fig. 7. Schematic of the idealized model. In two dimensions, we imagine a rectangular block of length \( L \) width \( 2R \) with \( L > R \). In three dimensions, we consider a disk of radius \( R \).](image-url)
a mean flow in this lubrication layer which may help or hinder the levitation. The most important effect however will be to drastically reduce the thermal conductivity between the block and the sand. For a completely levitating layer conduction would be almost zero since it could only occur against the motion of the sublimating gas. A turbulent lubrication layer could conduct some heat and radiative transfer might also be important. Most likely however the block will rise to the point where there is still sufficient solid contact to maintain a substantial heat flux and some intermediate state will be reached. Understanding these effects will be an important part of future work.

5.2. Checking the terrestrial experiments

For a block with $R = 100 \text{ mm}$ and on a terrestrial slope of $6^\circ$, $\tau = 0.62 \text{ s}$; on a slope at angle of repose, $\tau = 0.27 \text{ s}$. Using the values given in Table 2 for CO$_2$ blocks on the hot Navajo sand, we calculate $t^* > 5.2 \text{ s}$ for (a rectangular block and $H/R < 1$; $t^* > 1.5 \text{ s}$ for a disk)—a much larger value than $\tau$. For a block with $R = 100 \text{ mm}$, a minimum speed of $0.04 \text{ m s}^{-1}$ would be needed to maintain levitation. This is well below the observed velocities (the smallest measured average velocity was less than, but close to, $1 \text{ m s}^{-1}$ and the expected gravity-induced velocities ($7g\sin\theta$ ranges from $0.6 \text{ m s}^{-1}$ at $6^\circ$ to $1.4 \text{ m s}^{-1}$ at angle of repose). Thus, we are unsurprised that our dry ice blocks so easily slid down the terrestrial dune slopes.

5.3. Comparing Earth and Mars conditions

Using the values given in Table 2 for the CO$_2$ blocks on Mars, we estimate $t^* = 1.7 \text{ s}$ (for a disk) to $5.7 \text{ s}$ (for a rectangular block with $H/R = 1$) on Mars—a timescale comparable to the $t^*$ found for our terrestrial experiments. The timescale for block motion due to gravity, though, is slightly larger on Mars due to Mars' lower gravity: with $R = 100 \text{ mm}$, $\tau = 0.5 \text{ s}$ for slopes at angle of repose and $\tau = 1 \text{ s}$ for slopes at $6^\circ$ ($R = 100 \text{ mm}$). Observed blocks on Mars are a few meters in size, and this further increases $\tau$: with $R = 1 \text{ m}$, $\tau = 1.4 \text{ s}$ for slopes at angle of repose and $\tau = 3.2 \text{ s}$ for slopes at $6^\circ$.

This indicates that small blocks (of comparable aspect ratio) can be levitated as easily on Mars as they are on Earth and that large blocks are likely to also levitate. Sustained motion downslope for the large blocks may be a bit more difficult if they are in motion just due to the influence of gravity. When one includes the likely input of initial energy due to a block breaking and falling onto a dune slope, though, it seems likely that even the larger blocks could slide on Mars as easily as they slid on Earth.

This will hold true even at surface temperatures lower than the $T_0 = 260 \text{ K}$ used for our estimates. ACA-type gully activity observed in the southern polar pits is thought to have occurred when the surface temperature ranged from 190 to 260 K (Raack et al., 2012). Taking the lower-bound for $T_0$, we see that, as $t^* \sim (T_0 - T_1)^{0.5}$ (and other variables change only slightly at that temperature), $t^*$ decreases by maximally a factor of 10, making it slightly larger than, but still comparable to $\tau$ estimates. Thus, small blocks with initial velocity should still be mobilizable. In fact mobilization is theoretically possible at even lower temperatures: to support a $0.1 \text{ m}$ high block the sublimation pressure must exceed the atmospheric pressure by at least $p_{\text{ghl}} \approx 300 \text{ Pa}$; to generate this under a CO$_2$ ice block, the temperature needs to increase by only $3 \text{ K}$ (at 260 K the pressure generated will be more than 300 atmospheres).

An assumption of our model is that the gas can be treated as incompressible. This means that the relative excess pressure $p_j / p_0 - 1$ must not be too large, therefore it is only strictly applicable for blocks less than a certain height. The height of a levitating CO$_2$ block that would give an excess pressure average of $p_0$ on Mars is $p_0 / \rho_j g = 0.1 \text{ m}$. This means that for larger blocks a correct theory should include compressibility. We do not expect, however, that this would substantially change the conclusions of the theory.

5.4. Comparing CO$_2$ and H$_2$O sublimation on Mars

On Earth, as our experiments have demonstrated, H$_2$O ice block are not mobile on dunes since they do not sublimate. However, if a water ice block were to somehow form on Mars, can it then be mobilized via sublimation?

On the Earth, a water ice block could levitate above a surface above 100 °C by melting and then rapidly boiling since the vapor pressure is above atmospheric pressure—this is the Leidenfrost effect. On Mars, the situation is quite different. The sublimation pressure of H$_2$O is given for 130–273.16 K (T; the triple point of water) by Wagnera et al. (2011). They provide a formula of high accuracy but also show that the Clausius–Clapeyron sublimation law

$$p_s = p_0 e^{\frac{-E}{T_s}}$$

(10)

provides a very good approximation, where $p_0 = 612 \text{ Pa}$ the triple point pressure and $R_g = 463 \text{ J kg}^{-1} \text{ K}^{-1}$ the specific gas constant. Using this formula the partial pressure of H$_2$O at 147 K is 2.6 µPa—extremely low.

The atmosphere of Mars contains little H$_2$O, but since this pressure is so far below martian atmospheric pressure any sublimation will be very slow. As the partial pressure is low, there may in fact be equilibrium or deposition. If we assume that the H$_2$O block is at

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mars</th>
<th>Earth</th>
<th>Units</th>
<th>Description</th>
</tr>
</thead>
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<tr>
<td>$T_0$</td>
<td>260</td>
<td>300</td>
<td>K</td>
<td>Surface temperature</td>
</tr>
<tr>
<td>$p_0$</td>
<td>510</td>
<td>101,000</td>
<td>Pa</td>
<td>Atmospheric pressure</td>
</tr>
<tr>
<td>$g$</td>
<td>3.7</td>
<td>9.81</td>
<td>m s$^{-2}$</td>
<td>Gravity</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.013</td>
<td>0.26</td>
<td>W m K$^{-1}$</td>
<td>Sand thermal conductivity</td>
</tr>
<tr>
<td>$c$</td>
<td>680</td>
<td>830</td>
<td>J kg$^{-1}$ K$^{-1}$</td>
<td>Sand heat capacity</td>
</tr>
<tr>
<td>$k$</td>
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<td>10$^{-11}$</td>
<td>m$^2$</td>
<td>Sand permeability</td>
</tr>
<tr>
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<td>1540</td>
<td>kg m$^{-3}$</td>
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</tr>
<tr>
<td>$T_s$</td>
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<td>147</td>
<td>K</td>
<td>Ice block temperature</td>
</tr>
<tr>
<td>$e$</td>
<td>571</td>
<td>2830</td>
<td>kJ kg$^{-1}$</td>
<td>Volatile enthalpy of sublimation</td>
</tr>
<tr>
<td>$v$</td>
<td>13.2</td>
<td>13.2</td>
<td>Pa s</td>
<td>Volatile gas viscosity</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>0.010</td>
<td>0.0042</td>
<td>kg m$^{-3}$</td>
<td>Volatile gas density</td>
</tr>
<tr>
<td>$\rho_t$</td>
<td>1562</td>
<td>920</td>
<td>kg m$^{-3}$</td>
<td>Volatile solid density</td>
</tr>
</tbody>
</table>

Relevant physical quantities for martian Basaltic sand under martian conditions ($T_0 = 260 \text{ K}$ and $p_0 = 510 \text{ Pa}$ and dry Navajo Sand under Earth conditions ($T_0 = 293 \text{ K}$ and $p_0 = 101 \text{ kPa}$). Quantities for CO$_2$ are also specified for martian and Earth conditions. The properties of water/steam are taken from Wagner and Pruss (2002).
the same temperature as a CO2 block (and the CO2 frost covered martian dune surface), then it will be 147 K. The partial pressure at this temperature is less than 2 μPa. To generate the needed 300 Pa of excess pressure, the sublimation pressure must be 300 + 510 = 810 Pa which requires a temperature of 277 K. This is above the triple point of water so this would no longer be sublimation, but melting followed by vaporization—the classical Leidenfrost effect.

However, even if the sand is at this temperature the ice block will not rapidly produce water vapor until its surface temperature reaches 271 K, where the sublimation pressure exceeds atmospheric pressure. This means that there would be an initial phase where the ice block gradually warms by the heat flux from the sand and only much later would significant water vapor be produced. It is therefore more appropriate to use 271 K as the initial temperature of the water ice block. However, by the time of year that the block can reach this temperature, then realistic martian sand temperatures will not be much higher than this, so the expected temperature difference between block and sand will be far too small to give a large enough heat flux for levitation. Additionally, the initial phase of warming would not be sustained for a long period as the surface sand would be likely to cool down faster than the ice block could warm up.

Thus, while our model predicts that CO2 blocks can sublimate and move on both Earth and Mars, it shows that that water ice blocks are unlikely to mobilize via this mechanism on either planet.

6. Conclusion

We have presented an alternative hypothesis that explains the formation of martian linear gullies due to CO2 ice blocks breaking from the dune surface and then sliding downslope, carving out a track. Based on considerations of observed martian dune linear gully morphology, recent observations of CO2 ice/frost blocks within martian linear gullies and of groove formation/elongation, and terrestrial experiments and analogue features that form grooves via the downslope movement of blocks, we have clearly demonstrated the feasibility of this dry mechanism. Although our model and experiments do not (yet) quantitatively explain all characteristics of observed linear gullies (in particular, we have not observed well-developed meanders or large pit formation), it is far more consistent than water-based debris flow hypotheses with observed gully morphologies and activity and our understanding of the martian present-day environment. We now have a qualitative understanding of the conditions under which these features are likely to form: a dislodged block from a zone of high frost-accumulation falls onto dry, cohesionless material, and a temperature difference between the ice block and the sand surface then induces sufficient sublimation to levitate the block and move, via sliding and/or rolling, downslope. Our theoretical model of the levitation force generated by sublimation under a block also provides the beginnings of a quantitative understanding of the requisite environmental conditions for this mechanism to be active.

Future modeling should aim to improve our understanding of important role that sublimation plays in levitating and transporting these blocks, and in redistributing sand. If we can more fully quantify the requisite environmental conditions, then we can predict the timeframe in which we expect to see isolated blocks appear, gullies form and elongate, and pits form. These predictions can then be tested via comparison with observations, which will be acquired through a continued multi-temporal HiRISE monitoring campaign. Experiments run in a controlled and more Mars-like environment are also needed to derive quantitative relations between slab thickness, ice block characteristics, and gullies features.

Acknowledgments

Diniega was supported by an appointment to the NASA Postdoctoral Program, administered by Oak Ridge Associated Universities, at the California Institute of Technology Jet Propulsion Laboratory under a contract with NASA.

Appendix A. Flow solution

We non-dimensionalize the problem by scaling our coordinates by R, u by ut0 and scale p by Rvk/ut0. We take z ≥ 0 to be the region with the sand and work in two-dimensional cartesian or three-dimensional cylindrical coordinates so that |r| < 1 corresponds to the ice block. Taking the divergence of Eq. (1) and using incompressibility we then have

\[ \nabla^2 p = 0, \quad z > 0, \quad p|_{z=0} = 0, \quad |r| > 1, \quad \frac{\partial p}{\partial z}|_{z=0} = -1, \quad |r| < 1. \]

(A.1) (A.2) (A.3)

Thus we need to solve Laplace’s equation with mixed boundary conditions. In both cases, the solution is rather complicated to derive so we merely show the solution and demonstrate that it satisfies Eqs. A.1, A.2, A.3. The solutions are most conveniently expressed in complex variables, but only the real parts of p, ν1 and ν2 are physical. We define the complex variable \( w = r + iz \) and use the streamfunction ψ.

A.1. Two-dimensional result

The solution can be written

\[ \psi = iw + \sqrt{1 - w^2}, \quad v_1 = \frac{\partial \psi}{\partial w} = -i + \frac{w}{\sqrt{1 - w^2}}, \quad v_2 = -i \frac{\partial \psi}{\partial z} + 1 + \frac{iw}{\sqrt{1 - w^2}}, \quad p = iw + \sqrt{1 - w^2}. \]

(A.4) (A.5) (A.6) (A.7)

Since p is an analytic function of w for all z > 0 this satisfies Laplace’s equation and it is immediately apparent that \( v_2 = -\frac{dp}{dr} \) and \( v_1 = -\frac{dp}{dr} \).

On z = 0 we have

\[ p|_{z=0} = \begin{cases} \sqrt{1 - r^2}, & |r| < 1, \\ 0, & |r| > 1. \end{cases} \]

(A.8)

\[ v_{1|z=0} = \begin{cases} 1, & |r| < 1, \\ 1 - \frac{r}{\sqrt{1 - r^2}}, & |r| > 1. \end{cases} \]

(A.9)

\[ v_{2|z=0} = \begin{cases} \frac{r}{\sqrt{1 - r^2}}, & |r| < 1, \\ 0, & |r| > 1. \end{cases} \]

(A.10)

To get the total force on the block we integrate the pressure over \( r = -1 \cdots 1 \) and multiply by L.

\[ F = L \int_{-1}^{1} p(r, 0) dr = \frac{L \pi}{2}. \]

(A.11)

The streamlines and pressure distribution is shown in the upper part of Fig. 8. In the lower part of Fig. 8 p, \( v_1 \) and \( v_2 \) are plotted near the surface and far away. The very large values for the velocity near the surface at \( r = 1 \) are evident. This means that sand grains will always be mobilized at the edge of a sublimating block.
A.2. Three-dimensional result

The solution can be written as

\[
p = \frac{2}{\pi} \left[ iz \sin^{-1}\left( \frac{z+i}{r} \right) - \frac{i}{2} \sqrt{r^2 + (z+i)^2} \right],
\]

(A.12)

\[
\nu_z = \frac{2}{\pi} \left[ iz \sin^{-1}\left( \frac{z+i}{r} \right) + \frac{1}{2} \sqrt{r^2 + (z+i)^2} \right],
\]

(A.13)

\[
\nu_r = \frac{2}{\pi} \left[ \frac{i(r^2 + z^2) - z}{r} \right],
\]

(A.14)

\[
\psi = -\frac{1}{\pi} r \sin^{-1}\left( \frac{z+i}{r} \right) + (iz+1) \sqrt{r^2 + (z+i)^2}.
\]

(A.15)

These satisfy the Stokes equation and incompressibility and \( \psi \) is the stream function.

\[
\frac{\partial p}{\partial z} = \frac{1}{r} \frac{\partial \psi}{\partial r},
\]

(A.16)

\[
\frac{\partial \psi}{\partial r} = -\frac{1}{r} \frac{\partial p}{\partial z},
\]

(A.17)

\[
\nabla^2 p = \nabla \cdot \mathbf{v} = 0.
\]

(A.18)

It can be shown that \( p \) satisfies Laplace’s equation. When \( z = 0 \) we have

\[
p = \begin{cases} 
\frac{2}{\pi} \sqrt{1-r^2}, & |r| < 1, \\
0, & |r| > 0,
\end{cases}
\]

(A.19)

\[
\nu_z = \begin{cases} 
\frac{1}{\pi} \sin^{-1}\left( \frac{z}{r} \right) - \frac{1}{r} \sqrt{r^2-1}, & |r| < 1, \\
0, & |r| > 0,
\end{cases}
\]

(A.20)

\[
\nu_r = \begin{cases} 
\frac{2}{\pi} \sqrt{1-r^2}, & |r| < 1, \\
0, & |r| > 0.
\end{cases}
\]

(A.21)

The total force on the disc of material is

\[
F = \int_0^1 2 \frac{2}{\pi} \sqrt{1-r^2} \pi r dr = \frac{4}{3}.
\]

(A.22)

Again, both components of the velocity are infinite at the edge, suggesting that sand will always be mobilized. The form of the pressure and velocity on the surface is also remarkably similar to the two-dimensional case. The results are shown in Fig. 8.

Appendix B. Supplementary material

Supplementary data associated with this article can be found, in the online version, at [http://dx.doi.org/10.1016/j.icarus.2013.04.006](http://dx.doi.org/10.1016/j.icarus.2013.04.006).

References


